MATHCOUNTS® Minis November 2017 Activity Solutions

Warm-Up!

1. A number is divisible by 4 if the two-digit number formed by the tens and ones digits is a multiple of 4. Since we are looking for the greatest four-digit number, we should try to put the 7 and 5 in the thousands and hundreds places, respectively. Doing so leaves the 1 and 2 for the remaining places in the four-digit number. We are looking for the greatest number, so we first consider the number 7521, but the two-digit number 21 is not a multiple of 4. The two-digit number 12, however, is a multiple of 4, which leads to our answer, **7512**.

2. We want the number 481,5K6 to be divisible by 2, 3, 4 and 9. Let's take a look at the divisibility rules for 2, 3, 4 and 9:

Factor	Divisibility Rule
2	ones digit is 0, 2, 4, 6 or 8
3	sum of the digits is a multiple of 3
4	two-digit number formed by tens and ones digits is a multiple of 4
9	sum of the digits is a multiple of 9

The ones digit is 6, so this number will be divisible by 2 for any value of K. For this number to be a multiple of 3, however, we need the sum $4 + 8 + 1 + 5 + K + 6 = 24 + K$ to be a multiple of 3. If $K = 3$, the sum of the digits is $24 + 3 = 27$, which is a multiple of 3 and 9. That would ensure that the number is divisible by both 3 and 9. If $K = 3$, then the two-digit number formed by the tens and ones digits would be 36, which is a multiple of 4. So, the number 481,5K6 is divisible by 2, 3, 4 and 9 when K = **3**.

3. Since $2 + 3 + 7 + 9 = 21$, which is a multiple of 3, we know that each of the 24 four-digit positive integers that can be formed using each of the digits 2, 3, 7 and 9 will be a multiple of 3. Therefore, **0** of those integers are prime.

4. We are looking for a positive integer whose cube root is between $8^3 = 512$ and $8.1^3 = 531.441$. This integer also must be divisible by $18 = 2 \times 9$. We know that numbers that are divisible by 2 have a ones digit of 0, 2, 4, 6 or 8, and a number is divisible by 9 if the sum of its digits is a multiple of 9. Let's start at 512 and count up by 2s until we reach a number that is divisible by 9. We have 514, 516, 518 and 520, none of which are divisible by 9. Finally, we get to 522, which works since $5 + 2 + 2 = 9$. The answer is **522**.

The Problems are solved in the MATHCOUNTS® Mini video.

Follow-up Problems

5. Since the integer ages are for teenagers, we know the values of x, y and z are between 13 and 19, inclusive. We also know that $xyz = 4590$, which is a multiple of 5. Let's suppose that $x = 15$. Then $yz = 4590 \div 15 = 306$. Among the integers 13, 14, 16, 17, 18 and 19, is there a pair whose product is 306? Looking at the ones digits we see that 14 and 19 might work. Multiplying, we get $14 \times 19 = 266$, which doesn't work. Again, looking at the ones digits, we see that 17 and 18 might work. Multiplying, we get $17 \times 18 = 306$, which works. So if $x = 15$, $y = 17$ and $z = 18$, we get $x + y + z = 15 + 17 + 18 =$ **50**.

6. If we let m = ABCD, then the number formed by reversing the digits is DCBA. Both ABCD and DCBA are divisible by 45 = 5 × 9. Multiples of 5 have a ones digit of 0 or 5. Since both ABCD and DCBA are four-digit numbers that are divisible by 5, it follows that A = D = 5. So, we have 5BC5 and 5CB5. Both numbers must be divisible by 9, and we are told that m = 5BC5 is also divisible by 7. That means 5 + B + C + 5 = 10 + B + C must be a multiple of 9. Remember that we are looking for the greatest possible value of m, so let's start with 9 and 8 for B and C, respectively, since 10 + 9 + 8 = 27, which is a multiple of 9. Checking for divisibility by 7 yields 5985 ÷ 7 = 855. So, m = **5985**.

7. We are looking for the smallest five-digit number, with all different digits, that is divisible by each of its non-zero digits. The smallest possible five-digit number is 10000. Let's start with 10_ _ _ and try to minimize the other three digits. If we use the digits 2 and 3, we need to ensure that the number is divisible by 2, meaning it has a ones digit of 0, 2, 4, 6 or 8, and that is divisible by 3, meaning the sum of the digits is a multiple of 3. Starting with 1023_, the sum of the digits, so far, is 1 + 0 + 2 + 3 = 6, which is a multiple of 3. We need to minimize the value of the final digit, but it can't be 0 or 2 since those have already been used. Making the final digit 4 doesn't work for two reasons. First, 1 + 0 + 2 + 3 + 4 = 10, which is not a multiple of 3. And, second, 34, the two-digit number formed by the tens and ones digits, is not a multiple of 4. Making the final digit 6 does work, since 1 + 0 + 2 + 3 + 6 = 12, which is a multiple of 3. Also, a number is divisible by 6 if it is divisible by both 2 and 3, which is the case with **10,236**.

8. In order for this four-digit number to be divisible by 9, the sum of the digits must be a multiple of 9. We are looking for the smallest four-digit number that is divisible by 9 and that has two even digits and two odd digits. The smallest possbile four-digit number is 1000. Let's start with 10_ _. So far, we have 1 + 0 = 1. We won't be able to find a pair of digits, one even and one odd, whose sum is 8 because and even plus and odd is always odd. So, let's try to find a pair of digits, one even and one odd, whose sum is 17. The only pair that works is 8 and 9. Thus, the smallest four-digit number that works is **1089**.